LESSON 7·3

Fractions and Percents of a Set

How do we find a fraction or percent of a number (a set)?

Students make rectangular arrays with 24 blocks. They identify the factors and fractional parts of the number 24. Next, students use 10%, 5%, and 1% to quickly identify any percent of 24. Finally, they discuss computational methods for finding fractions and percents of a set.

Objectives

- To identify factors and fractional parts of a number
- To use 10%, 5%, and 1% to quickly identify percents of any number
- To find fractions and percents of a set by converting to decimals and multiplying

Materials

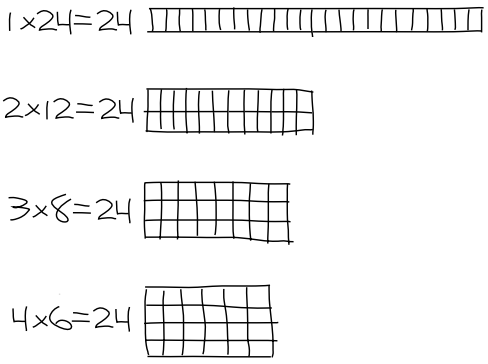
Each group will need:

- 60 single blocks
- tenth and hundredth blocks
- 1 decimal mat

- 1 calculator **per student**
- 1 *Factors and Fractions* activity sheet **per student**
- 1 blank piece of paper
- 1 *Tip the Waiter* activity sheet **per student**

◆ Class Introduction 30 MIN.

- Distribute single blocks to each group. Have them count out 24 blocks and make a rectangular array with their blocks. Ask each group to identify their array as the number of rows × the number of columns. Have students quickly sketch each distinct array on the board or chart paper.



Review how each of these arrays represents the various factors for the number 24. List the factors:

Factors of 24
1, 2, 3, 4, 6, 8, 12, 24

- Next, have all the groups make an array with 2 rows of 12. Ask,

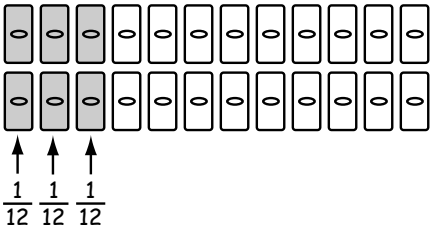
How can I use the array to find 1/2 of 24? *(Identify 1 out of 2 equal parts - that's one of the 2 rows. There are 12 blocks in each row, so 1/2 of 24 = 12.)*

How can I use the array to find 1/12 of 24? *(Identify 1 out of 12 equal parts - that's one of the 12 columns. There are 2 blocks in each column, so 1/12 of 24 = 2.)*

How can I use the array to find 2/12 of 24? *(Identify 2 out of 12 equal parts - that's two of the 12 columns. There are 4 blocks in each column, so 2/12 of 24 = 4.)*

Continue identifying fractions of the set of 24. Explain that the × symbol can be read as "of" and write these as multiplication sentences:

$^3/_{12}$ of 24	**→**	**$^3/_{12} \times 24 = 6$**
$^4/_{12}$ of 24	**→**	**$^4/_{12} \times 24 = 8$**



- Next, ask students to consider 10% of 24 and 1% of 24. Write each case as a multiplication sentence, then discuss the steps in converting it to a familiar problem with decimals.

10% of 24	**→**	**10% × 24**	**→**	**0.1 × 24 = 2.4**
10% of 24 = 2.4				
1% of 24	**→**	**1% × 24**	**→**	**0.01 × 24 = 0.24**
1% of 24 = 0.24				

Have students represent 24, 2.4 and 0.24 with blocks on their decimal mats. Explain that just as they built 12/12 of 60, 100 groups of 0.24 would represent 100/100 of 24.

Finding 10% and 1% should be very easy. We simply shift the digits one place or two places smaller. It's the same as multiplying by 0.1 and 0.01 or dividing by 10 or 100.

Ask,

Now that we know 10% and 1% of 24, how can we find 5% of 24?
(Take half of 10% → 1/2 of 2.4 = 1.2. Or, take 5 of the 1% → 5 x 0.24 = 1.2)

Have students find other percents of 24, using combinations of 10%, 5% and 1%. Explain that this method for finding percents allows them to calculate percents mentally. For example:

17% of 24	**= 10% + 5% + 1% + 1%** **= 2.4 + 1.2 + 0.24 + 0.24** **= 4.08**
31% of 24	**= 10% + 10% + 10% + 1%** **= 2.4 + 2.4 + 2.4 + 0.24** **= 7.44**

- Finally, discuss computational methods for finding fractions and percents of a set. Explain that students can always convert the fraction or percent to a decimal and then multiply.

Have students try several problems. For example:

Elle made $47 delivering bagels in the neighborhood. She must give 2/5 of her earnings to the bagel store to pay for the bagels. How much does she owe the bagel store today?

$\frac{2}{5}$ of $47 → **$\frac{2}{5} \times 47$**

$\frac{2}{5} = 2 \div 5$

$\frac{2}{5} = 0.4$

→ **0.4×47**

= **18.8**

Elle owes the bagel store $18.80.

Samuel answered 95% of the questions on the test correctly. There were 60 questions. How many questions did Samuel answer correctly?

95% of 60 → **$95\% \times 60$**

$95\% = 0.95$

→ **0.95×60**

= **57**

Samuel answered 57 questions correctly.

◆ Group Activity 20 MIN.

- Distribute the activity sheets and blank paper. On the first activity sheet, *Factors and Fractions*, students are directed to make rectangular arrays for the number 60 and to find fractional parts of the arrays. They also calculate fractional parts by converting the fractions to decimals and then multiplying. Explain that they can draw and label the arrays on the blank piece of paper.

  On the second activity sheet, *Tip the Waiter*, students help New York City diners calculate tips. Remind students that it is easy to calculate percents if they use benchmark percents: 10%, 5%, and 1%. However, they may solve the problems any way they wish.

◆ Closure 10 MIN.

- As a class, list all the factors of 60. If there is time, have student volunteers draw the arrays and shade the specified parts. Discuss how to use a computational method to find these fractions of the number 60.
- Next, have students share their methods for calculating tips. Compare computational methods and mental methods.

◆ Assessment

- Do students use arrays to identify all the factors of a number?
- Can students identify fractional parts of an array?
- Can students quickly identify 10%, 5%, and 1% of any number? Do they use them to find composite percents of a number?
- Can students use a computational method to calculate fractions of a set? Percents of a set?

Name

Factors and Fractions

1. Make as many different rectangular arrays as you can. Each array should have exactly 60 single blocks. Quickly sketch each distinct array on a blank sheet of paper.

2. List the factors of 60:

3. Shade one of your arrays to show: $^2/_5$ of 60

   Shade one of your arrays to show: $^1/_{15}$ of 60

   Shade one of your arrays to show: $^2/_3$ of 60

   Shade one of your arrays to show: $^3/_{10}$ of 60

   Shade one of your arrays to show: $^7/_{30}$ of 60

4. Convert the fractions to a decimal, then multiply. Show your work.

   Example: $^2/_5 \times 60 = 24$ Here's how: $^2/_5 = 2 \div 5$

   $2 \div 5 = 0.4$

   $0.4 \times 60 = 24$

   $^1/_4 \times 60 =$ _____

   $^3/_{10} \times 60 =$ _____

   $^2/_5 \times 60 =$ _____

   $^7/_8 \times 60 =$ _____

Name

Tip the Waiter

In New York City, waiters make 20% for great service, 15% for average service, and 10% for poor service. Help these diners calculate the tips to give their waiters.

1. Dai has received great service. He wants to leave a 20% tip for the waiter.

   How much should Dai leave? Explain.

2. Andrea has very good service. She wants to leave a 17% tip for the waiter.

   How much should Andrea leave? Explain

3. Blanca has average service. She wants to leave a 15% tip for the waiter.

   How much should Blanca leave? Explain.

4. Stan has poor service. He wants to leave a 10% tip for the waiter.

   How much should Stan leave? Explain.