Place Value as a Red Herring: How to “Invent” Positional Notation

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Traditional Approach

- Counting View
  - One-by-one progression of the numbers
    - Odometer (non-digital) shows each turn of the number

      1
      2
      3
      .
      .
      .
      099
      100
      101
      .
      .
      .
      263

Traditional Approach

- Place Value

  Reinterpret the digits

  263 = 2 x 100 + 6 x 10 + 3 x 1
- Power View

  View Ones, Tens, Hundreds, … as “things”

  263 = 2**H** 6**T** 3**O**
- Advantages of the power view

  [2**H** 6**T** 3**O**] + [2**H** 3**O**] = 4**H** 6**T** 6**O**

- There are many opportunities for children to get lost along the way
- The pattern of the count is not obvious (0-9 not 1-10)

7
8
9
10

But not

20**7**
20**8**
20**9**
20**10**

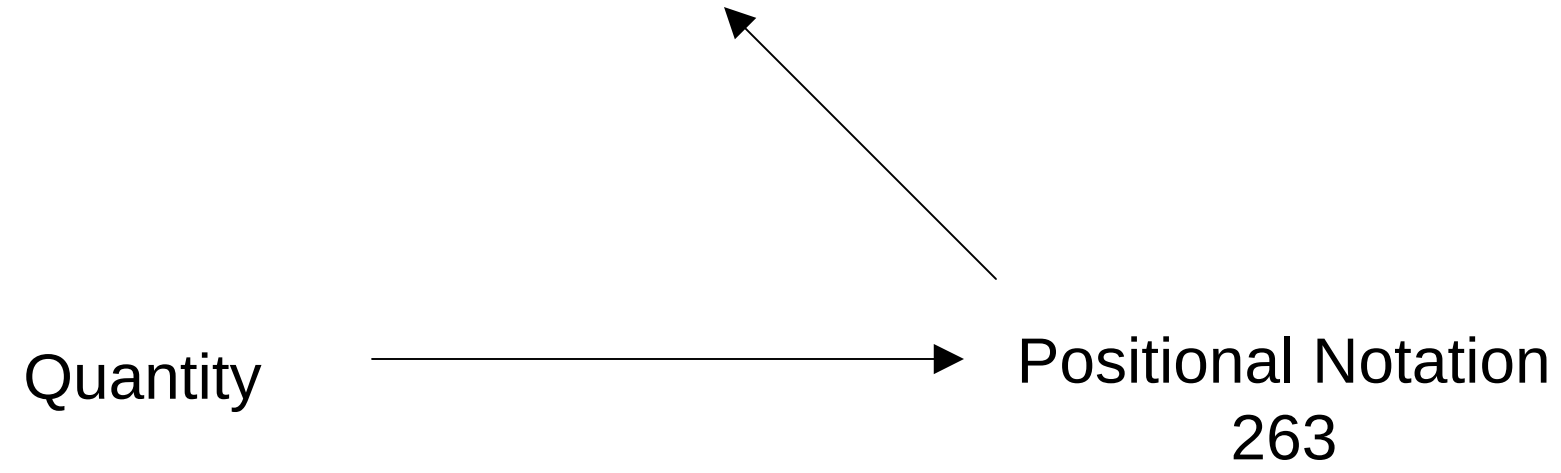
- The place-value lesson relies on knowledge of addition and multiplication

  $263 = 200 + 60 + 3 = 2 \times 100 + 6 \times 10 + 3 \times 1$

Traditional Approach

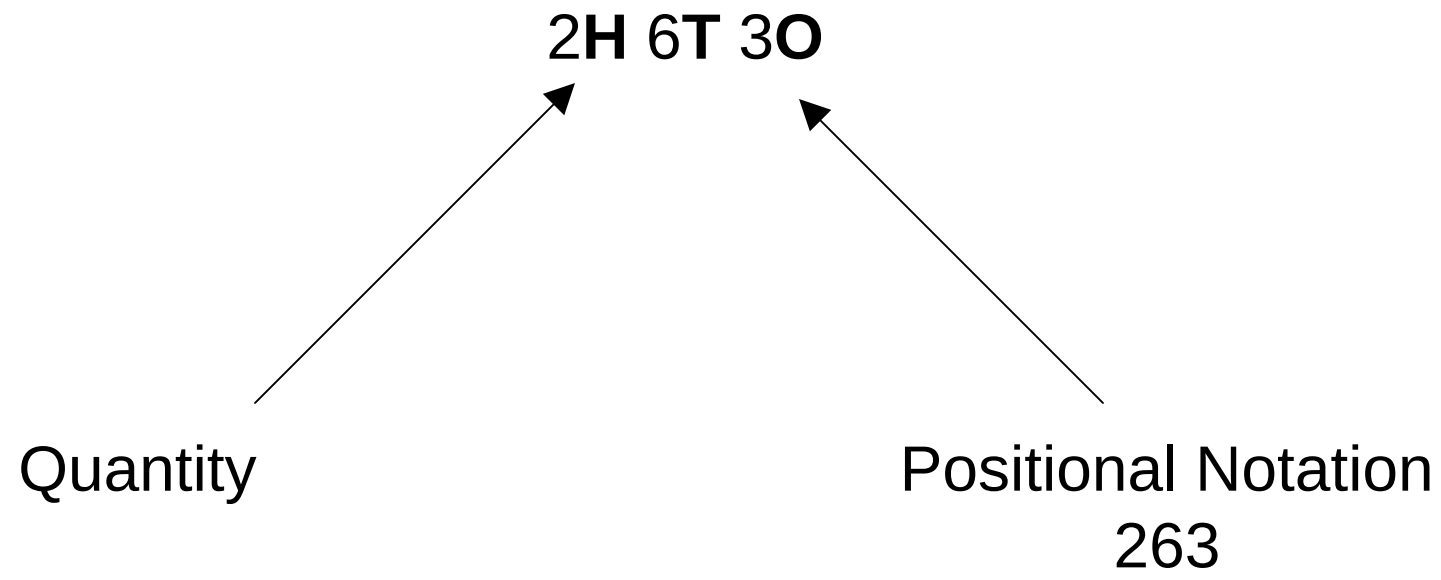
Organization in Ones, Tens, Hundreds, …

2**H** 6**T** 3**O**



Alternative Approach

Organization in Ones, Tens, Hundreds, …



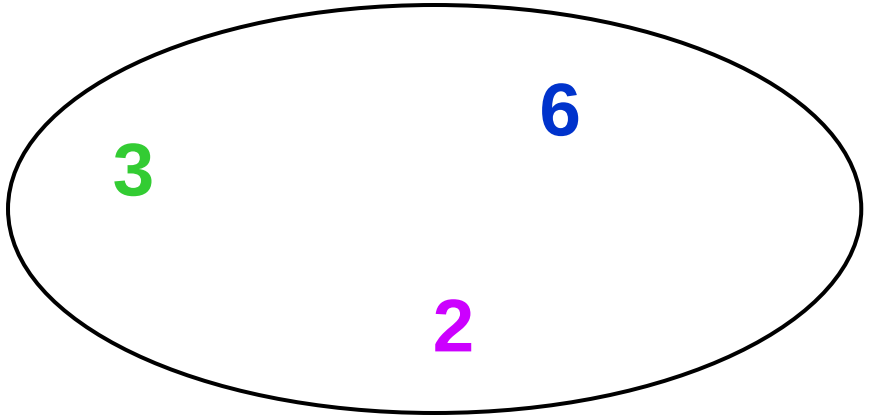
Alternative Approach

- Learn the digits

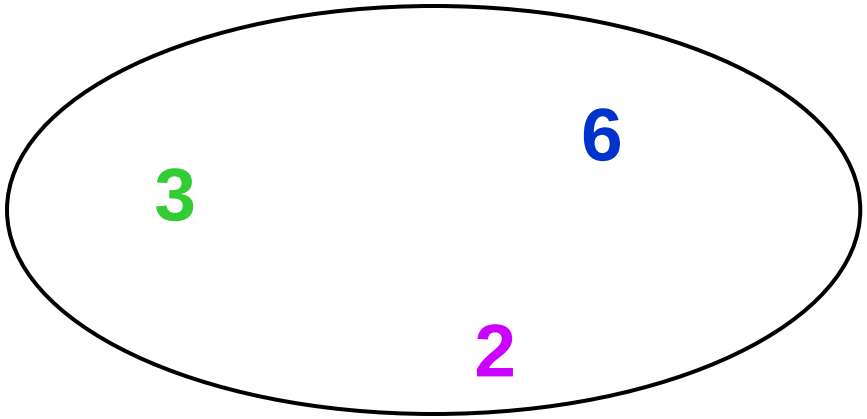
  1, 2, …, 9
- Organize any given collection of blocks into Ones, Tens, Hundreds, … with no more than nine of each

Alternative Approach

- Write a digit (1-9) to show how many of each size block (i.e., each power of ten)



(= 263)


6
3
2

Key Question

- If we wish to use the same digits (same size, same color) for every power-of-ten, how can we tell which digit relates to which size?
- There are many possible solutions

  e.g. $5\mathbf{T}^3\ 4\mathbf{T}\ 3 = 5{,}043$

  Positional notation with zero as a place-holder is beautiful and ingenuous but it is not the only way

3

2

Advantages of the Approach

- Fewer chances to get lost
- Positional notation arises on a solution to a problem
- Zero is not a mystery but a clever device

Advantages of the Approach

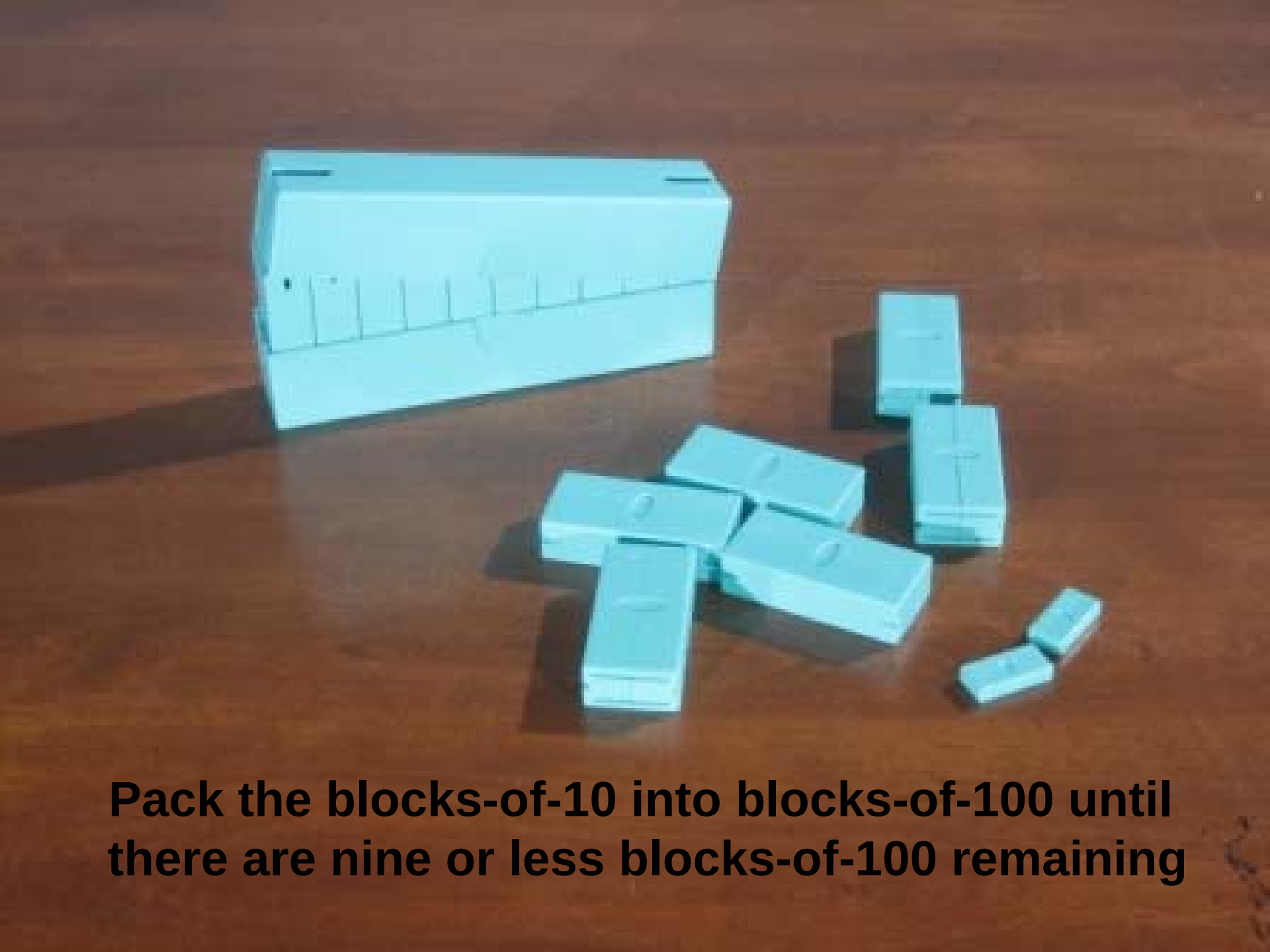
Key for making the approach workable:

A direct way of organizing a quantity (of blocks) in powers of ten.

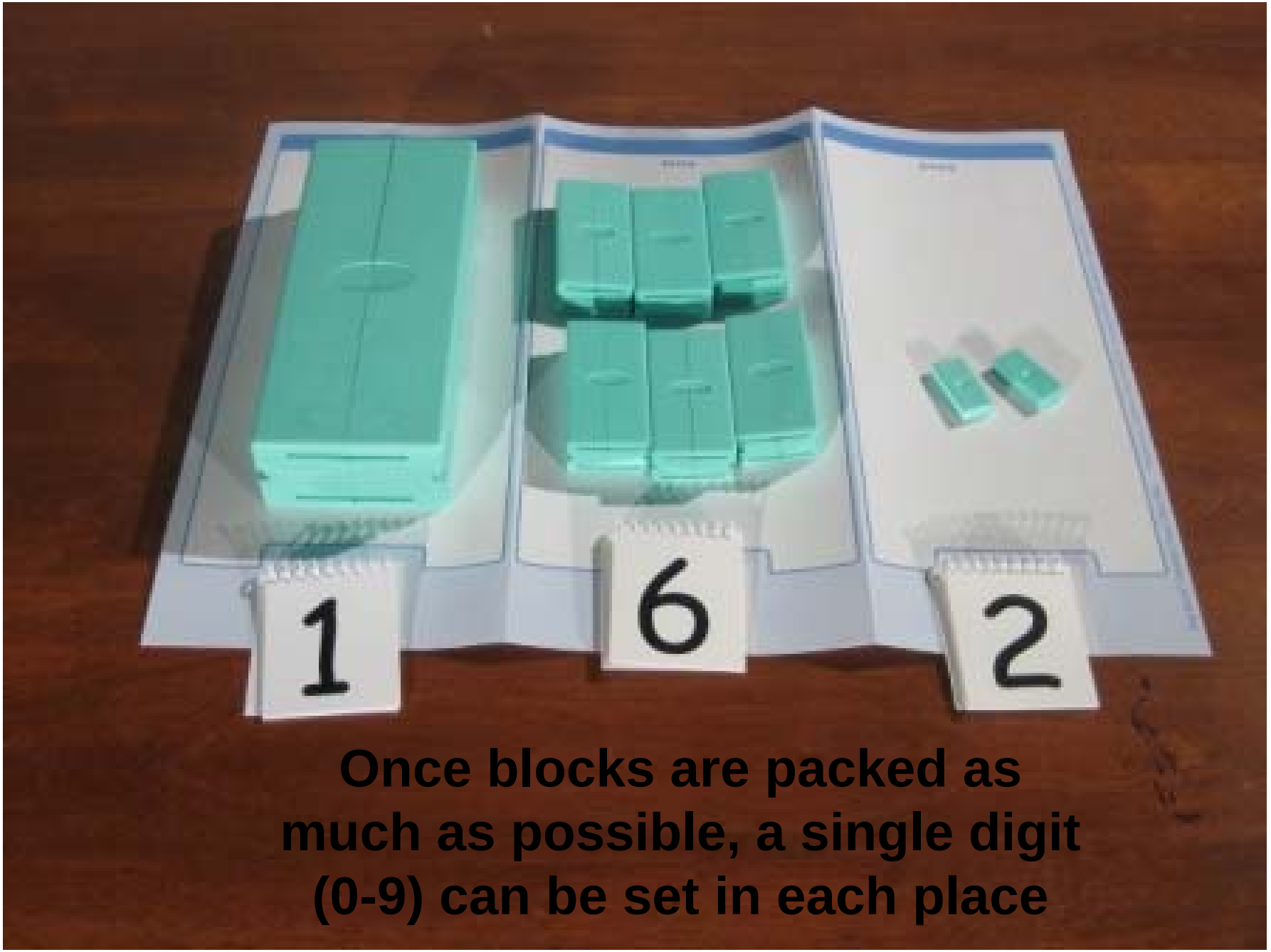
- This was the motivation for developing Digi-Block
- Ten singles pack to make a block of ten
- Ten tens pack to make a block of one hundred
- Need only a single instruction:
  - **Pack as much as possible!**

Take a collection of blocks

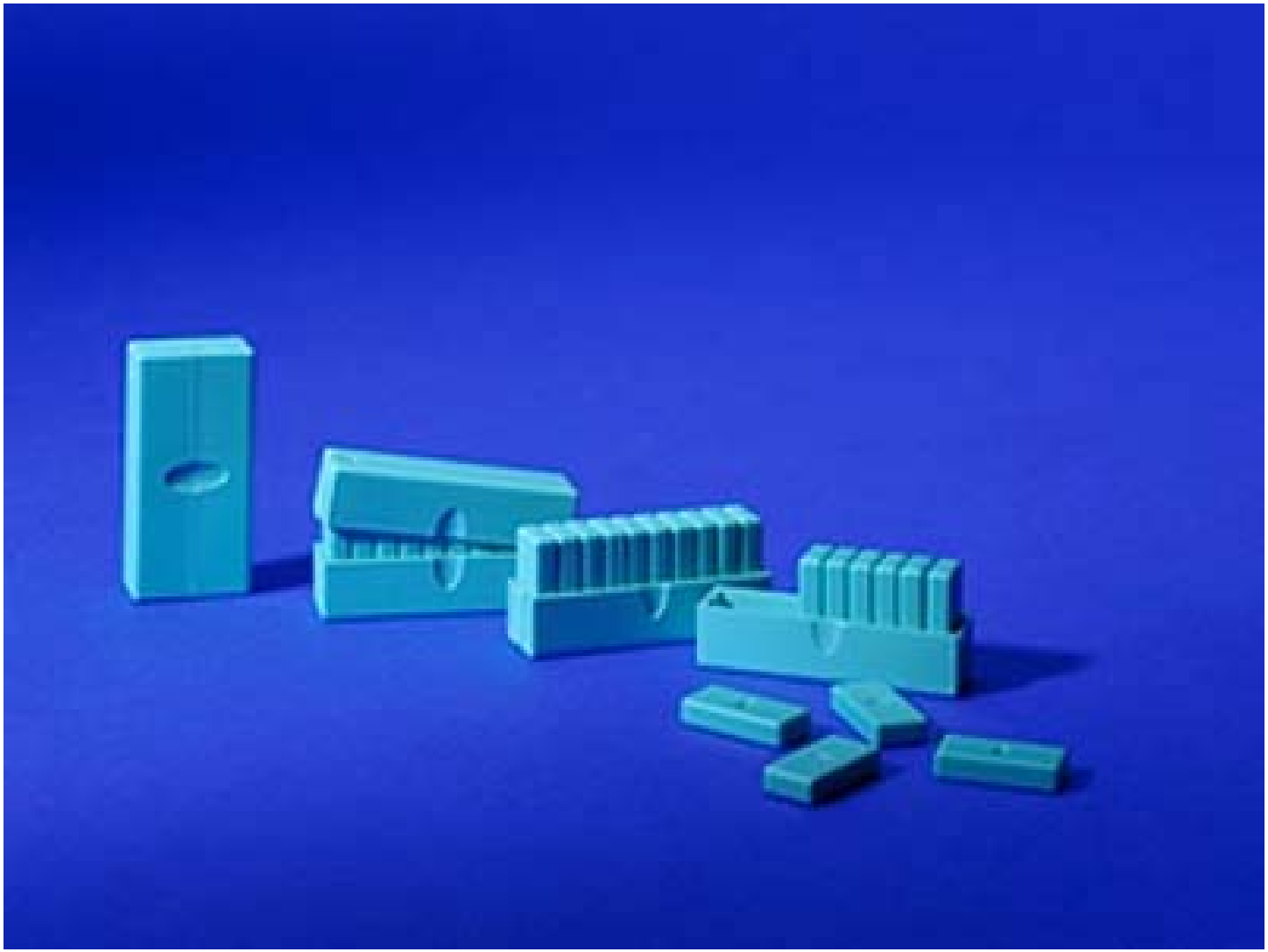
Pack the single blocks into blocks-of-10
Until there are nine or less blocks remaining

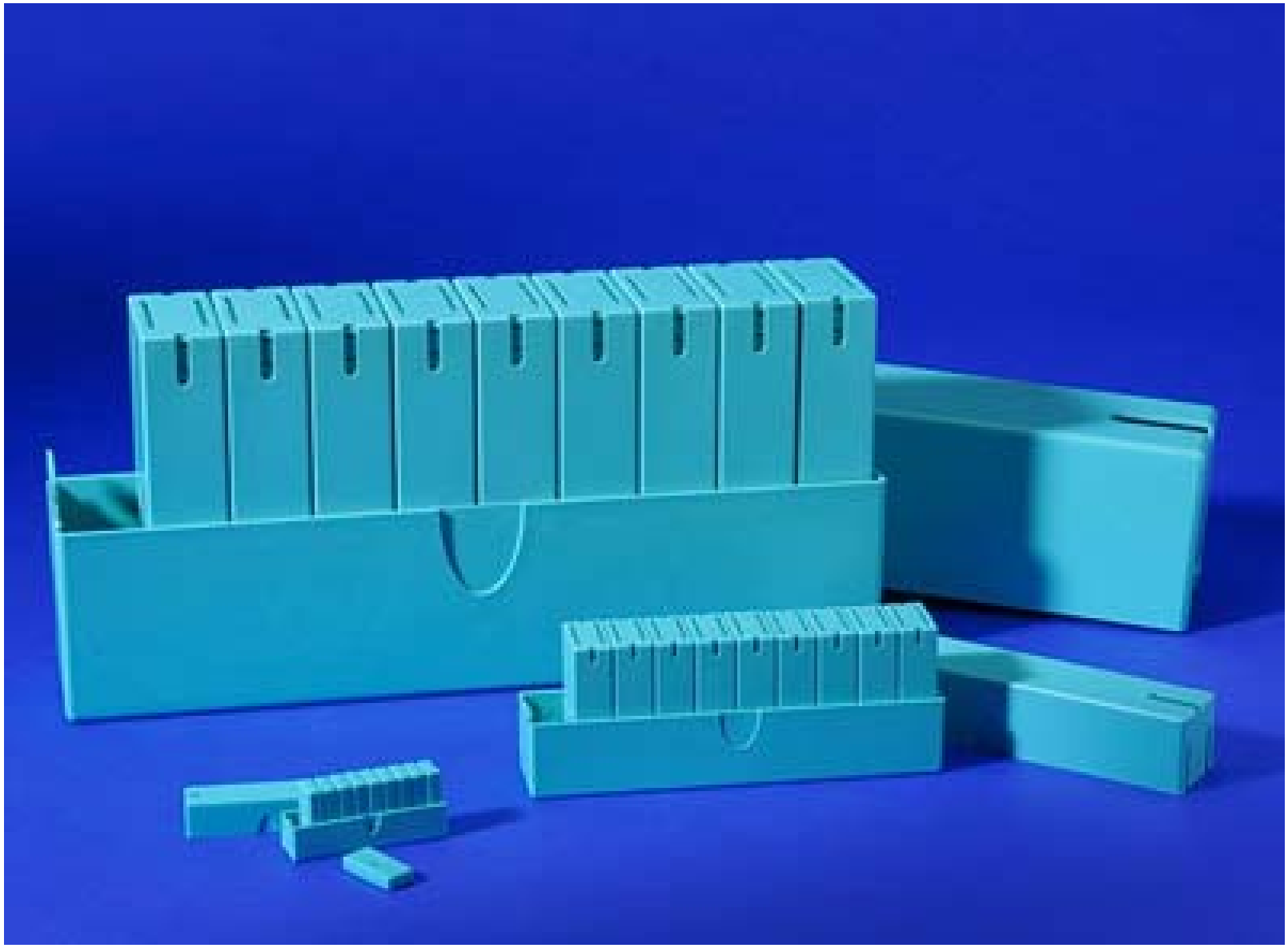


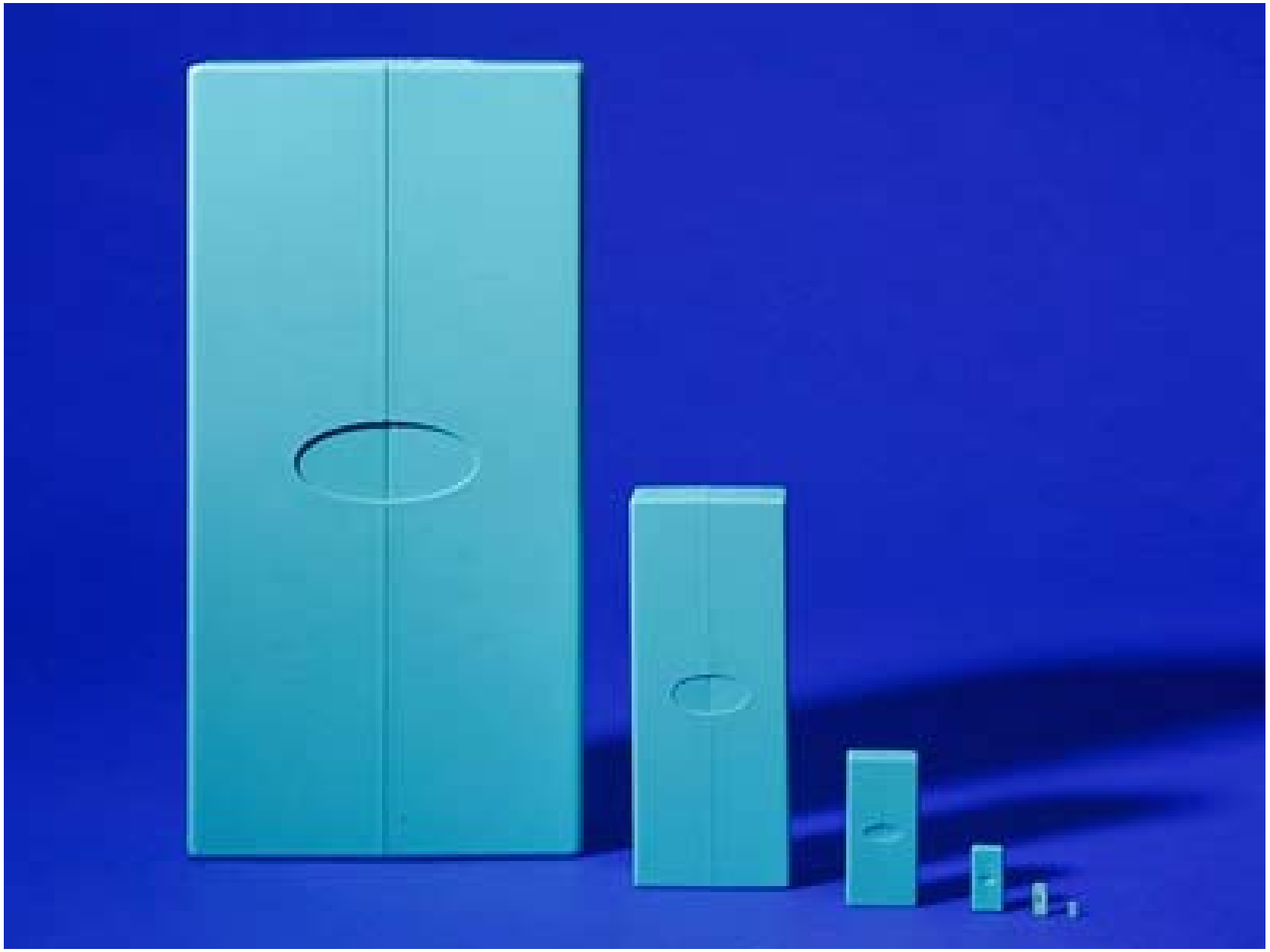
Pack the blocks-of-10 into blocks-of-100 until there are nine or less blocks-of-100 remaining

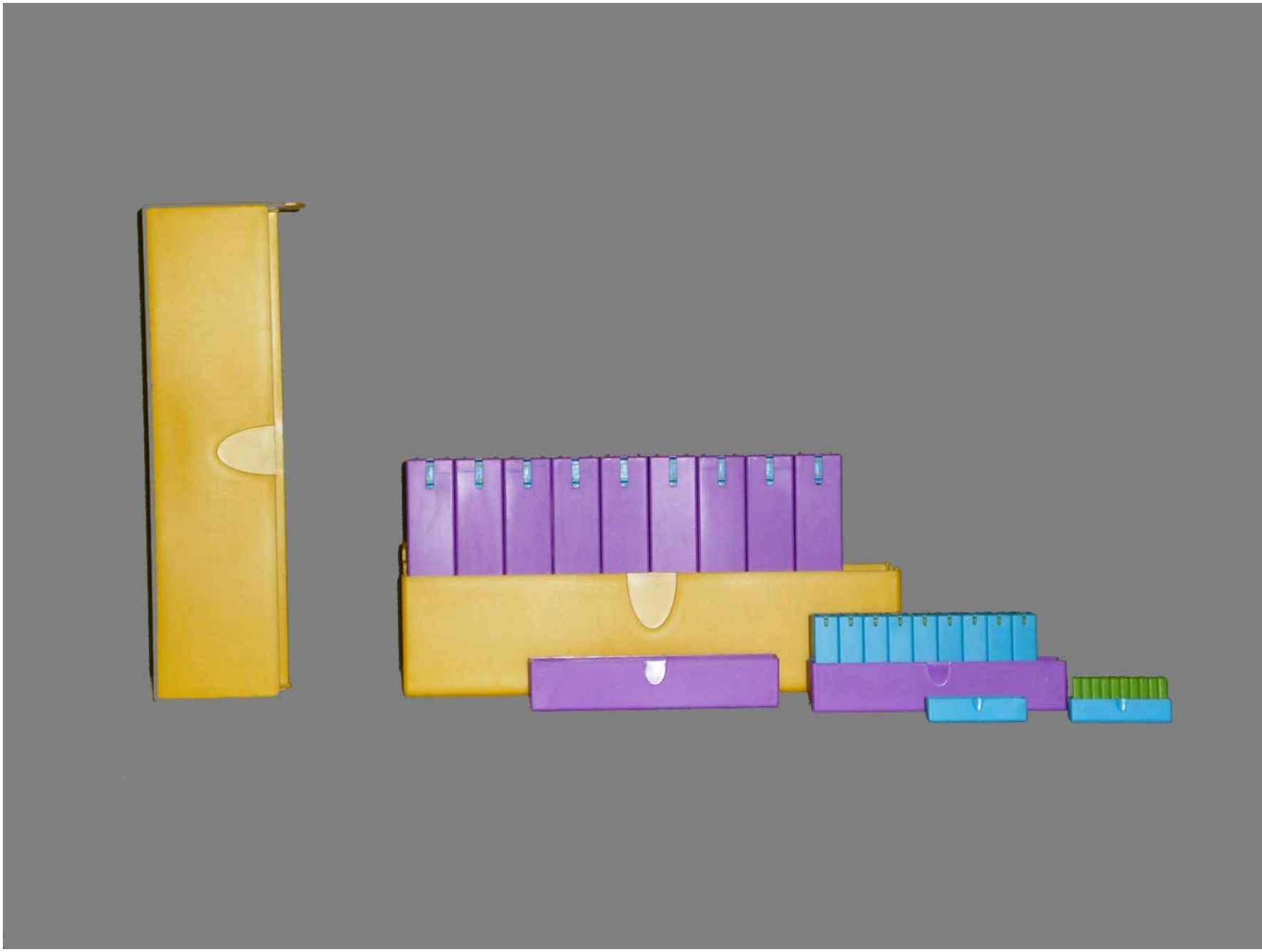


Once blocks are packed as much as possible, a single digit (0-9) can be set in each place









The progression shown in bright colors blocks

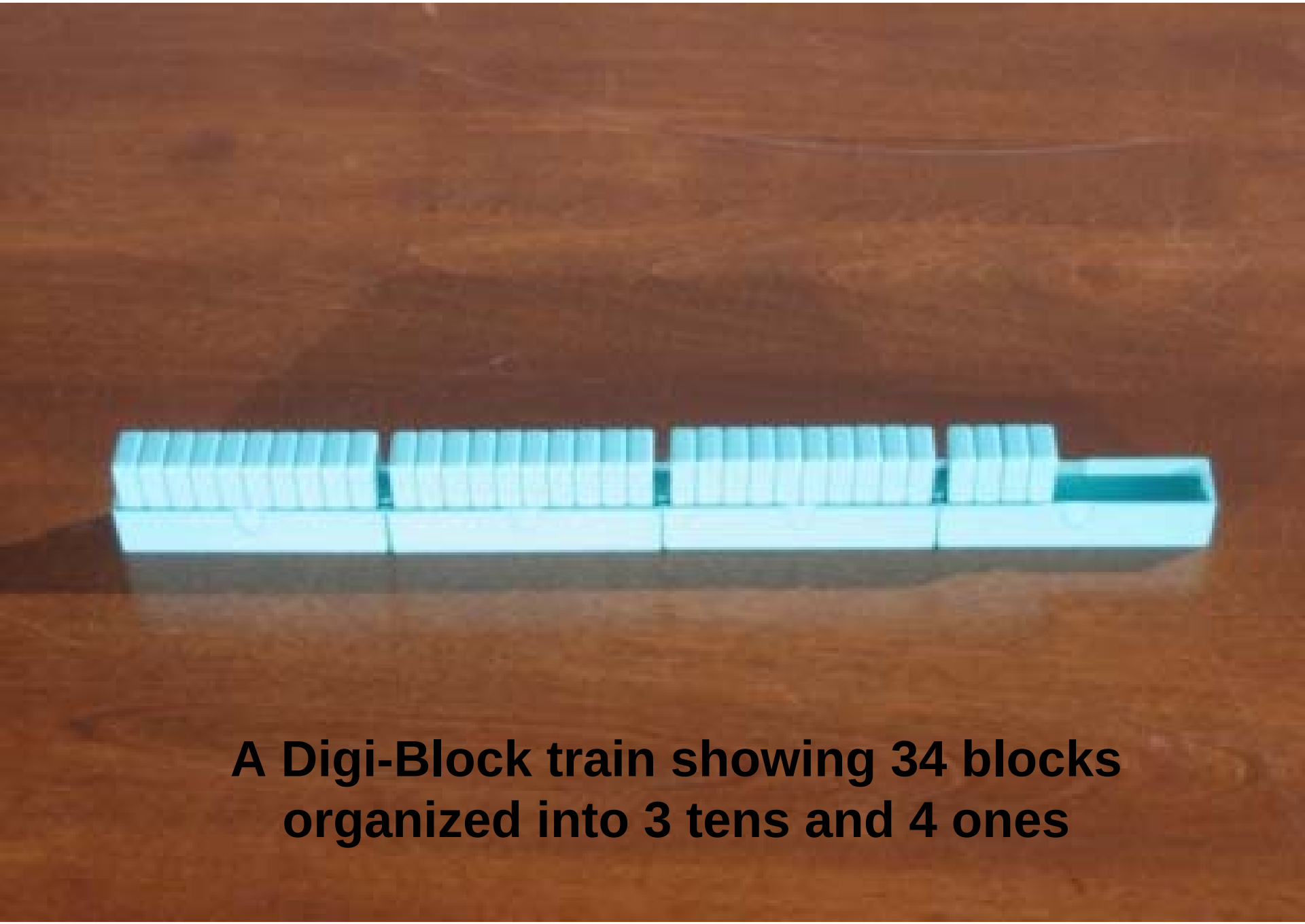
Thousands

Hundreds

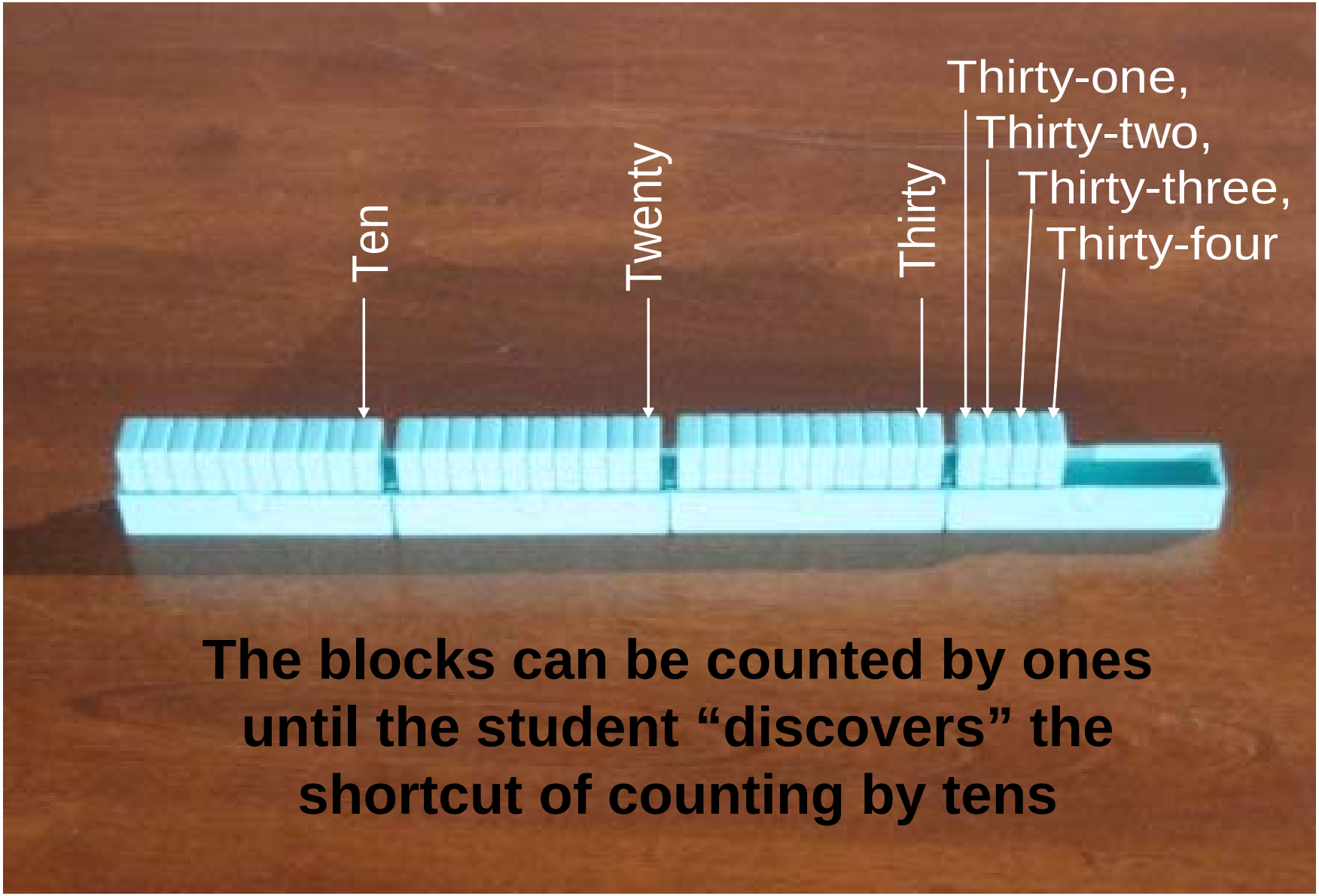
Tens

Ones

Can be represented by color rather than position at young ages



A Digi-Block train showing 34 blocks organized into 3 tens and 4 ones



The blocks can be counted by ones until the student “discovers” the shortcut of counting by tens

Presentation will be posted on

www.digi-block.com